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ANALYSIS OF EJECTOR THRUST BY INTEGRATION OF CALCULATED SURFACE PRESSURES

By John C. Sanders and Virginia L. Brightwell

Lewis Flight Propulsion Laboratory
Cleveland, Ohio

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SUMMARY

An analysis of the thrust of air ejectors was made for the purpose of discerning the nature of ejector action and the source of its thrust. Thrust force is computed by integration of pressures over the ejector surfaces. The pressures are determined by one-dimensional-flow analysis. Mixing of fluid streams is considered to proceed to completion. The equations are presented in a form that makes clear the nature of the thrust rather than in a form that lends itself to ease of use. In the analysis, the case of ejectors handling incompressible fluids is developed and some illustrative problems for the case of the ejector handling air considered as a compressible fluid are solved.

The analysis showed that ejector augmentation of the thrust generated in a primary-jet passage operating with incompressible fluids is created largely by pressure forces on the surfaces of the converging secondary passage preceding the mixing zone. These pressure forces, caused by the static-pressure rise that results from the mixing of fluid streams of different velocities, decrease with increase in secondary velocity.

INTRODUCTION

Understanding of air-ejector performance is currently incomplete, as evinced by the fact that many of its characteristics have not been explained. For example, a decline in ejector thrust with increase in simulated flight speed has been shown by an investigation reported in reference 1; a later analysis (reference 2), however, indicated that thrust gains resulting from the use of the air ejector may reappear at supersonic flight speeds. An increase in thrust augmentation caused by an ejector with increase in altitude is shown in reference 3. Static sea-level investigations of a rocket showed that no appreciable gain in thrust resulted from use of an air ejector. An appreciable amount of apparently contradictory data on the performance of air ejectors therefore exists.

Considerable progress has recently been made toward a more thorough understanding of the ejector. In an analysis of ejector performance (reference 4) assuming incompressible fluids, it was possible to account theoretically for the observed decline in thrust augmentation that accompanies increase in flight speed. Even these studies do not make current knowledge sufficiently complete for an understanding of the observed characteristics of the ejector. These studies contain mathematical operations too complex for easy interpretation in terms of physical conditions. Furthermore, they fail to give clues as to the nature of the force created by the ejector.

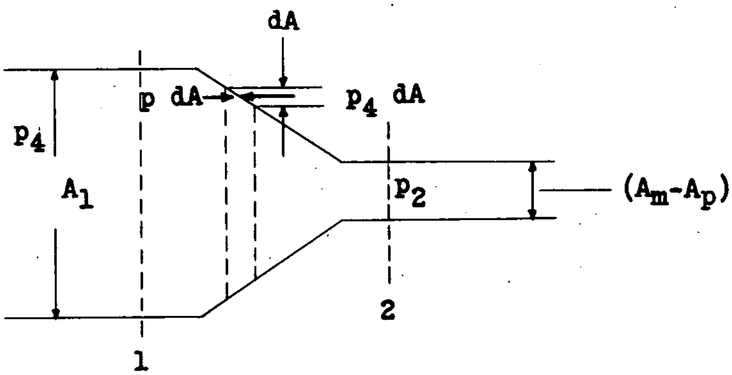
An analysis, contained herein, was made at the NACA Lewis laboratory to describe the force created by the ejector and to help explain some of the ejector characteristics. This analysis is based upon an integration of the pressures over the surfaces of the ejector and of the primary-jet passage. The primary-jet fluid is assumed to originate in the primary-jet passage, thus representing a rocket or a jet engine in which the augmentation is on the basis of jet thrust. Incompressible fluids are assumed to make the mathematics simple enough that the physical meanings of the operations may be easily traced. Analysis of the nature of ejector thrust is followed by an investigation of mixing-pressure rise and the resulting thrust augmentation. Finally, a discussion is presented of the modifications to the analysis necessary to account for compressibility of the fluid, together with examples of the difference in performance attributable to this compressibility.

NATURE OF EJECTOR THRUST WITH INCOMPRESSIBLE FLUID

Description of ejector and definition of terms. - The air-ejector configuration used for analysis of ejector characteristics with incompressible fluids is shown in figure 1. The primary jet engine generates its own primary fluid at a primary total pressure P_p and discharges this fluid through a nozzle having a cross-sectional area A_p. The ejector consists of a converging passage into which secondary fluid is inducted from the surrounding atmosphere and accelerated before entering the mixing zone at station 2. Complete mixing of the primary and secondary fluids is assumed to take place in the cylindrical mixing zone of cross-sectional area A_m between stations 2 and 3. After mixing, the fluid passes through a diffuser between stations 3 and 4 and is discharged at ambient pressure p_4. (All symbols are defined in appendix A.)

The mixing of the primary and secondary streams causes a rise in pressure (p_3-p_2) from station 2 to station 3. When a simple ejector without exit diffuser is considered, p_3 equals p_4, and the pressure difference (p_3-p_2) depresses the pressure p_2 at the exit of the primary nozzle below the ambient pressure p_4. The following analysis of ejector thrust is based upon an inspection of the effects of this reduction in pressure at station 2 on the pressures over the surfaces of the components of the ejector and the primary jet engine.

Axial force on secondary passage. - The axial thrust on the ejector was found by computing the pressures over the surfaces of a converging passage, surrounded by an incompressible fluid at ambient pressure p_4 and having a discharge pressure p_2. The following sketch shows the forces and the pressures involved:



The area at the exit of the converging passage is given as (A_m-A_p), the difference between area of the mixing zone and area of primary-jet nozzle, which obstructs the mixing zone passage at this point.

The assumption that pressure over the external surface of the secondary passage is the ambient static pressure p_4 is also adopted by inference in analyses in which the axial force is calculated by evaluation of the change in momentum through the propulsion system. This assumption therefore makes the present analysis by pressure integration comparable with the analysis of reference 4. If the angle of convergence is small, the assumption is reasonable for a nonviscous fluid.

The axial thrust upon an elemental frontal area dA of the ejector is shown to be

$$dF_e = (p_4-p)\ dA$$

where p is the internal pressure on the elemental area.

Details of the integration of this expression to give the force when integrated between entrance and exit areas and of finding the areas required to give a specified pressure differential (p_4-p_2) are presented in appendix B. These operations result in the following expression for the thrust on a converging passage:

$$\frac{F_e}{(P_s-p_4)(A_m-A_p)} = \left[1 - \left(1 + \frac{p_4-p_2}{P_s-p_4}\right)^{\frac{1}{2}}\right]^2 \tag{1}$$

Equation (1) expresses the thrust F_e of the secondary duct in terms of a thrust coefficient $\frac{F_e}{(P_s-p_4)(A_m-A_p)}$ involving the flight velocity head (P_s-p_4) and the area (A_m-A_p) of the secondary passage at the entrance of the mixing zone. The area at the entrance of the secondary passage A_1 has disappeared from the equation because A_1 was so adjusted that secondary fluid could be scooped in without spillage over the entrance edge. In other words, A_1 is the annular area of the ambient stream that is swallowed by the secondary passage. This area is therefore uniquely related to the areas and the pressures given in equation (1).

Forces on primary-jet passage. - The ejector lowers the pressure at the exit of the primary jet from the ambient pressure p_4 to the pressure at the entrance to the mixing zone p_2, and changes the pressures on the primary-jet passage. In order to determine the thrust of the primary-jet passage, it is therefore necessary to integrate the pressures over the surfaces of the primary-jet passage in a manner similar to that used in the determination of the thrust on the secondary passage. This derivation is presented in appendix C. The resulting equation for force on the primary-jet passage is

$$F_p = 2(P_p-p_4)\ A_p + (p_4-p_2)\ A_p \tag{2}$$

Equation (2) shows that the thrust of a primary-jet passage with a maintained total propellant pressure P_p and immersed in a fluid at static pressure of p_4 is increased by a reduction in the discharge pressure p_2. Conversely, if the discharge pressure is increased, the thrust is decreased.

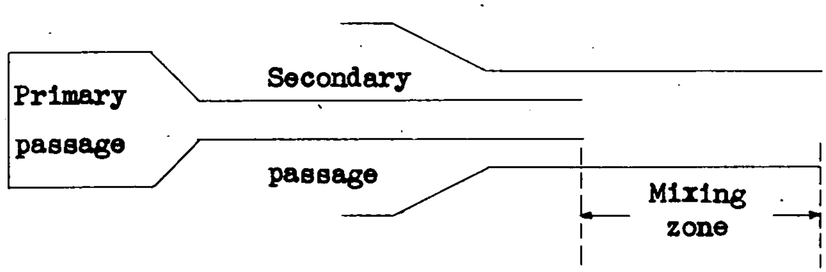
Axial forces on exit diffuser. - Equation (1) expresses the thrust F_e on the portion of the ejector ahead of the mixing zone. Another axial force F_d results from pressures over the surfaces of the diffuser or nozzle following the mixing zone. This force can be found by expressing the general equation (1) with symbols applicable to the diffuser between stations 3 and 4.

$$\frac{-F_d}{(P_3-p_4)\ A_m} = \left[1 - \left(1 + \frac{p_4-p_3}{P_3-p_4}\right)^{\frac{1}{2}}\right]^2$$

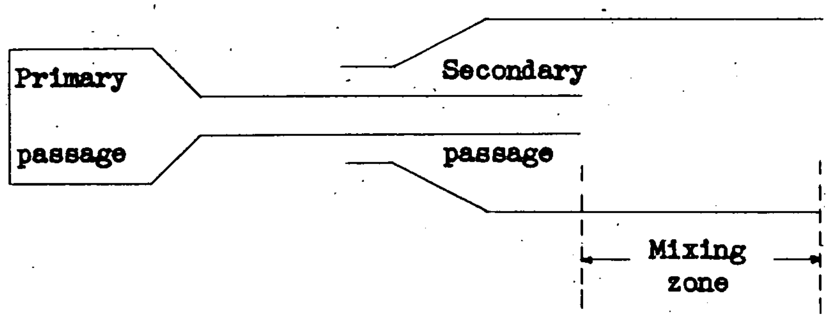
This equation can be rearranged to express F_d as a coefficient that can be added to the thrust coefficient of equation (1).

$$\frac{-F_d}{(P_s-p_4)(A_m-A_p)} = \left(\frac{A_m}{A_m-A_p}\right)\left(\frac{P_3-p_4}{P_s-p_4}\right)\left[1 - \left(1 + \frac{p_4-p_3}{P_3-p_4}\right)^{\frac{1}{2}}\right]^2 \tag{3}$$

Inspection of equation (1) reveals that the thrust coefficient of the secondary passage is a function of the ratio of the pressure rise (p_4-p_2) in mixing and diffusion to the flight velocity head (P_s-p_4) and is positive for both positive and negative values of the pressure rise. When the pressure rise is positive, the secondary passage converges ahead of the mixing zone, as shown in the following sketch:



On the other hand, when the pressure rise is negative, the secondary passage diverges ahead of the mixing zone:



Similarly equation (3) shows that the force on the diffuser or the nozzle at the exit of the mixing zone is always a drag, regardless of whether a nozzle or diffuser is used. Nozzles and diffusers at the exit of the mixing zone afford a possibility of regulating the mixing process to the extent that an increased mixing-pressure rise (p_3-p_2) might be achieved. This possibility is discussed later in this analysis.

Net thrust of propulsive system. - The integration of pressures over the surfaces of the primary passage and of the ejector (secondary passage) has shown that a deviation in pressure at station 2 (fig. 1), the beginning of the mixing zone, from the ambient pressure produces a force on the ejector and a change in the thrust of the primary-jet passage. If p_2 is depressed below ambient pressure p_4 by a pressure rise in the mixing zone, the thrust of the primary passage is increased and is therefore added to the ejector thrust to obtain the over-all increase in thrust. If, on the other hand, as a result of the mixing process and the expansion process a pressure p_2 is created that is greater than ambient pressure p_4, the thrust of the primary-jet passage is decreased and this decrease must be subtracted from the ejector thrust.

An equation expressing these variations in the net increase in thrust on the propulsion system is derivable from equations (1) to (3) and is as follows:

$$\frac{F_e + F_d + \Delta F_p}{(P_s-p_4)(A_m-A_p)} = \left\{1 - \left[1 + \left(\frac{p_4-p_2}{P_s-p_4}\right)\right]^{\frac{1}{2}}\right\}^2 - \left(\frac{A_m}{A_m-A_p}\right)\left(\frac{P_3-p_4}{P_s-p_4}\right)\left[1 - \left(1 + \frac{p_4-p_3}{P_3-p_4}\right)^{\frac{1}{2}}\right]^2 +$$

Secondary passage   Exit diffuser

$$\left(\frac{A_p}{A_m-A_p}\right)\left(\frac{p_4-p_2}{P_s-p_4}\right) \tag{4}$$

Primary nozzle

The terms representing the contributions of the components of the propulsive system are identified.

In the case in which the diffuser-pressure rise (p_4-p_3) is zero, all negative values of pressure rise (p_3-p_2) produce negative values of increase in thrust, $F_e + F_d + \Delta F_p$. For this case, the conclusion is drawn that a net increase of thrust is produced only when positive rise in pressure occurs in the mixing zone. Analysis of the mixing-pressure rise will be necessary to determine (P_3-p_4), after which a comparison will be made of thrust with positive and negative values of mixing-pressure rise in an ejector in which the diffuser-pressure rise (p_4-p_3) is not zero.

PRESSURE RISE IN MIXING ZONE

Fundamental momentum equation. - The pressure rise that occurs during the mixing of two parallel streams in a passage of constant cross-sectional area may be computed from a fundamental equation that relates pressure changes and momentum changes between two stations. This fundamental equation is

$$A_m\, p_3 - A_m\, p_2 = \rho_p\, A_p\, V_p^{\,2} + \rho_s(A_m-A_p)\, V_s^{\,2} - \left[\rho_p\, A_p\, V_p + \rho_s(A_m-A_p)\, V_s\right]V_3 \tag{5}$$

The left-hand member of this equation represents the force balancing the change in momentum denoted by the right-hand member. The first term of the right-hand member represents the momentum of the primary stream; the second term, the momentum of the secondary stream; and the third term, the momentum of the fluids at the exit to the mixing zone when the portion of this term in brackets is the mass of fluid exhausted.

The velocity at the exit of the mixing zone V_3 may be computed for incompressible fluids by the following equation:

$$V_3 = \frac{A_p}{A_m} V_p + \frac{(A_m-A_p)}{A_m} V_s \tag{6}$$

Equation (5) now reduces to

$$\frac{p_3-p_2}{\rho_p V_p^{\,2}} = \frac{A_p}{A_m} + \frac{\rho_s}{\rho_p}\frac{(A_m-A_p)}{A_m}\left(\frac{V_s}{V_p}\right)^2 -$$

$$\left[\frac{A_p}{A_m} + \frac{\rho_s}{\rho_p}\frac{(A_m-A_p)}{A_m}\left(\frac{V_s}{V_p}\right)\right]\left[\frac{A_p}{A_m} + \frac{(A_m-A_p)}{A_m}\left(\frac{V_s}{V_p}\right)\right] \tag{7}$$

Equation (7) describes the pressure rise (p_3-p_2) in mixing. From this equation, however, it is difficult to perceive how the pressure rise will be affected by changes in primary and secondary velocities, areas, and densities. The following inspection of these equations was therefore made by concentrating the analysis on each of the variables.

<u>Mixing-pressure rise for density ratio of 1.</u> - Investigation of the effect of the velocity ratio V_s/V_p is simplified by assuming the densities of the primary and secondary streams to be equal. Equation (7) reduces to

$$\frac{p_3-p_2}{2(P_p-p_2)} = \frac{A_p}{A_m}\left(1 - \frac{A_p}{A_m}\right)\left(1 - \frac{V_s}{V_p}\right)^2 \tag{8}$$

In equation (8) the effects of velocity ratio and primary- and mixing-passage areas are apparent. The greatest pressure rise (p_3-p_2) is achieved with the secondary velocity equal to zero. An increase in secondary velocity decreases the mixing-pressure rise. At a secondary velocity equal to the primary velocity, the mixing-pressure rise becomes zero. This characteristic is shown in figure 2. At any specified area ratio A_p/A_m, an increase in velocity ratio decreases the mixing-pressure-rise parameter $\frac{p_3-p_2}{P_p-p_2}$.

The effect of area ratio A_p/A_m is also shown in figure 2. As the ratio of the area of the primary stream to that of the secondary stream is reduced from unity, the pressure rise at any velocity ratio increases to a maximum at an area ratio A_p/A_m of 0.5; a further decrease in this area ratio decreases the pressure rise until, at an area ratio of 0, the pressure rise has dropped to 0.

Effect of density ratio on mixing-pressure rise. - The effect of density ratio ρ_s/ρ_p on the pressure rise in mixing is not easily deduced by examination of equations (5) and (7) because this ratio appears in opposed terms in equation (7), and because it bears an indirect influence through its effect on the relation between the the velocity ratio and the ratio of velocity heads in the secondary and primary streams $\frac{P_s-p_2}{P_p-p_2}$. Collecting all terms involving density in equation (7) yields

$$\frac{p_3-p_2}{P_p-p_2} = 2\frac{A_p}{A_m}\left(1-\frac{A_p}{A_m}\right)\left\{\frac{P_s-p_2}{P_p-p_2}-\left[\left(\frac{\rho_s}{\rho_p}\right)^{\frac{1}{2}}+\left(\frac{\rho_p}{\rho_s}\right)^{\frac{1}{2}}\right]\left(\frac{P_s-p_2}{P_p-p_2}\right)^{\frac{1}{2}}+1\right\} \tag{9}$$

The densities appear in one simple and understandable term in equation (9). An increase in the density term $\left[\left(\frac{\rho_s}{\rho_p}\right)^{\frac{1}{2}}+\left(\frac{\rho_p}{\rho_s}\right)^{\frac{1}{2}}\right]$ simply reduces the pressure rise in mixing. Inspection of the density term shows that it has a minimum value with a density ratio ρ_s/ρ_p of 1.0. Either an increase or a decrease in density ratio from this value of 1.0 increases the density term, as shown in figure 3, and decreases the pressure rise in mixing. In fact, a specified density ratio will give the same pressure rise as the reciprocal value of density ratio.

Combined effects of density ratio and velocity ratio on mixing-pressure rise. - Presentation of the effects of both velocity ratio and density ratio on mixing-pressure rise would be confusing in a figure such as figure 2 because an additional variable is added. Figure 2 has therefore been converted to a three-dimensional plot represented by oblique projection in figure 4. Only area ratios A_p/A_m below 0.5 are shown in figure 4 because the area ratios above 0.5 are not of such great interest in thrust augmentation, and the figure is simplified by this omission. The information omitted is, however, symmetrical with that shown and the information presented in this figure is the same information as that shown in figure 2 with the addition of surfaces representing performance at density ratios of 3 and 10. The velocity ratio has been replaced by the equivalent velocity-head ratio. The surfaces shown give the mixing-pressure rise as a function of area ratio A_p/A_m and of velocity-head ratio $\frac{P_s-p_2}{P_p-p_2}$.

In figure 4 the great effect of density ratio on the mixing-pressure rise is apparent. The density ratio plays a particularly important role in limiting the maximum value of velocity-head ratio at which pressure rise can be achieved. Mathematical proof can be developed to show that the velocity-head ratio at which the pressure rise becomes zero is equal to the density ratio or to the reciprocal of the density ratio. Such a relation is revealed by an inspection of figure 4. The illustrative density ratios of 3 and 10 were chosen as representative of the density ratios encountered in the application of ejectors to jet engines and rockets, respectively.

THRUST AUGMENTATION OF PRIMARY JET

Definition of augmentation. - All the information needed for computing augmentation has been presented. The ejector, primary, and diffuser thrusts may be computed from equations (1) to (3) in terms of mixing-pressure rise, and the mixing-pressure rise may be computed by equation (9) for specified values of primary and secondary pressures and densities. A precise and useful definition of augmentation is needed before this calculation is made. The definition used in this analysis is expressed by the equation

$$S = \frac{(F_e + F_d + F_p)/W_p}{F_{p,f}/W_{p,f}} \tag{10}$$

Equation (10) was used to compute the augmentation obtainable with various values of ram parameter $\frac{P_s-p_4}{P_p-p_4}$ and density ratio $\frac{\rho_s}{\rho_p}$. This new ram parameter is different from the velocity-head ratio used in figure 4, but the mixing-pressure rise is computable from equation (9) using this parameter because

$$\frac{P_s-p_4}{P_p-p_4} = \frac{\left(1 - \frac{p_4-p_2}{P_s-p_2}\right)}{\left(\frac{P_p-p_2}{P_s-p_2} - \frac{p_4-p_2}{P_s-p_2}\right)}$$

where

$$\frac{p_4-p_2}{P_s-p_2} = \frac{p_4-p_3}{P_s-p_2} + \frac{p_3-p_2}{P_s-p_2}$$

Solutions are first made for the case of no pressure change through the exit diffuser, that is, $\frac{p_4-p_3}{P_s-p_2}$ equals zero. The effect of exit-diffuser action is then investigated by assigning values of $\frac{p_4-p_3}{P_s-p_2}$.

Augmentation without exit diffuser. - Figure 5 shows the decrease in augmentation that accompanies increase in ram pressure (P_s-p_4) revealed in other analyses and experiments. In this figure, the area-ratio scale has been reversed from that of figure 4 because the augmentation is shown to increase with decrease in the area ratio A_p/A_m, although there is an opposite trend in mixing-pressure rise. The reason for this reversal is that the ejector thrust is proportional to the secondary area. Decreasing the area ratio increases the secondary area more rapidly than it decreases the pressure rise.

As would be expected from analysis of the mixing-pressure rise, a departure of the density ratio from unity decreases the augmentation. If the ram parameter $\frac{P_s-p_4}{P_p-p_4}$ is greater than the value of the density ratio or its reciprocal, the ejector creates a reduction in thrust rather than an augmentation. In the case of a jet engine in which density of the primary fluid is 0.3 times that of the atmosphere, a ram parameter greater than 1.2 would result in drag on the ejector. In the case of the rocket with a density ratio of 10, a similar case exists.

Effect of exit diffuser or nozzle on augmentation. - Comparison of ejector augmentation with and without diffuser or nozzle is made on the assumption that a limitation on the maximum diameter of the diffuser or mixing zone exists. For an illustrative case, the ratio of the primary-nozzle area to the maximum area of the diffuser or nozzle A_p/A_{max} was assigned a value of 0.15. Then for various values of the nozzle- or diffuser-area ratio A_4/A_m, the augmentation was computed. Under these conditions, values of A_4/A_m less than 1.0 represent nozzles, and variation in the ratio A_4/A_m caused no change in the ratio of primary-nozzle area to mixing-zone area. Values of A_4/A_m above 1.0 represent diffusers, and because the exit of the diffuser is larger than the mixing zone, and because the largest area of the diffuser is of specified size relative to the primary nozzle, then increases in the diffuser-area ratio A_4/A_m represent decreases in the mixing-zone area as expressed by the ratio A_p/A_m.

The change in augmentation (referred to augmentation obtained with A_4/A_m of 1.0) caused by the nozzle or diffuser is shown in figure 6. The curve for A_4/A_m of 1.0 represents the ejector without nozzle or diffuser. The nozzle, represented by A_4/A_m of 0.8, reduces the augmentation at all values of the ram parameter except 1.0. The diffuser, represented by A_4/A_m of 2.0, 2.4, and 3.0, shows increases in augmentation. Increasing the diffuser increases the augmentation to a finite limit, however, and further increase in diffuser-area ratio decreases the augmentation.

One of the purposes in investigating the diffuser was to see if the loss in augmentation with increase in ram parameter that was shown in figure 5 could be eliminated. Figure 6 shows that although the loss in augmentation with increase in ram parameter is less for a diffuser-area ratio A_4/A_m of 2.4 (approximately the most favorable value of A_4/A_m) than for the ejector alone, this loss is still quite severe, dropping from 1.54 at a ram parameter of 0 to 1.08 at a ram parameter of 0.3. The possibility of avoiding the loss in ejector thrust that accompanies increase in flight speed by controlling velocity-head ratio $\frac{P_s-p_2}{P_p-p_2}$ through the use of an exit diffuser or nozzle is therefore precluded.

Summary of analysis of ejector with incompressible fluid. - The analysis of the flow of an incompressible fluid through an ejector has shown that the thrust force causing augmentation arises from a drop in static pressure from the inlet of secondary fluid to the beginning of mixing. This decrease in static pressure at the inlet of the secondary passage is accompanied by a pressure rise in mixing. Increasing the flight speed increases the secondary velocity at the entrance to the mixing zone, thereby decreasing the pressure rise in the mixing zone and decreasing the thrust-creating pressure drop in the secondary fluid. The analysis has also shown that an exit diffuser or nozzle is ineffective in preventing this loss in thrust with increase in flight speed and that the greatest thrust is obtained with primary and secondary fluids of equal density.

Utility of ejector as a pump. - Figures 4 and 5 show that if the total pressure of the secondary stream is low relative to that of the primary stream, both an increase in mixing-pressure rise and, consequently, in thrust are obtained by the installation of an ejector. If some unavoidable loss in secondary pressure were incurred, the ejector would afford a greater increase in thrust at high flight speeds. Such a phenomenon occurs in the case of the boundary layer over the external surfaces of aircraft. The ejector can provide the pumping action necessary for removing low-energy air from the surfaces, and at the same time increase the thrust of the primary engine. This gain may be realized at high flight speeds. The gain, however, does not compensate for the over-all loss resulting from incurring the loss in the boundary layer. The ejector may be used to recover part of this loss without impairing the thrust of the primary jet engine.

EFFECT OF FLUID COMPRESSIBILITY

Scope of investigation. - Essential differences in the performance of ejectors handling compressible fluids as compared with ejectors handling incompressible fluids result from differences in the effect of pressures on the thrust of a duct handling fluids and from the difference in pressure rise with mixing of compressible fluids as compared with incompressible fluids. The reversal in the relation between passage area and fluid velocity in passing through sonic velocity results in the possibility of many more different configurations of the primary engine and ejector for compressible fluids than would be used with incompressible fluids. The complexity of relations among passage area, pressure, velocity, and temperature in the flow of a compressible fluid makes more difficult the application of generalized equations for the performance of ejectors. An additional variable, the ratio of static pressure to total pressure, must be considered. Consequently, the analysis of the flow of compressible fluids presented herein was executed by trial-and-error solution and by numerical integration, and only sample solutions are presented. Solution was made for an ejector operating with a ratio of densities of the primary and secondary streams at the beginning of mixing of 1.0. The nozzle for the primary stream was assumed to be shaped properly to discharge the primary fluid at the pressure of the secondary fluid at the beginning of mixing. Only one ratio of primary-nozzle area to mixing-zone area A_p/A_m was used. This area ratio was small (0.0625) and was chosen to represent an ejector that would provide a large thrust augmentation.

Force on converging duct. - The method for computing the force on the ejector passage handling a compressible fluid is basically the same as in the case of the incompressible fluid except that the pressures are related for isentropic flow of gas. The force is given by integration of the pressures as indicated in the following equation:

$$\frac{F}{P_s(A_m-A_p)} = \int_{(A_m-A_p)}^{A_1} \left(\frac{p_1}{P_s} - \frac{p}{P_s}\right) \frac{dA}{(A_m-A_p)} \tag{11}$$

where from equations of isentropic flow of gases

$$\frac{A}{(A_m-A_p)} = \left[\frac{\left(\frac{p_2}{P_s}\right)^{\frac{2}{\gamma}} - \left(\frac{p_2}{P_s}\right)^{\frac{\gamma+1}{\gamma}}}{\left(\frac{p}{P_s}\right)^{\frac{2}{\gamma}} - \left(\frac{p}{P_s}\right)^{\frac{\gamma+1}{\gamma}}}\right]^{\frac{1}{2}} \tag{12}$$

The forces were obtained by numerical integration of equation (11) and the results are shown in figure 7, together with thrust produced by an incompressible fluid. It may be seen that in an incompressible fluid, a decrease in p_2 below p_1 always increases the thrust. With the compressible fluid, however, a decrease in p_2 increases the thrust until the value of p_2/P_s equivalent to sonic velocity is reached. Further reduction in p_2 reduces the thrust and eventually causes a drag. It is therefore concluded that a compressible fluid always gives less thrust at a specified pressure difference (p_1-p_2) than does an incompressible fluid. These trends may be applied to the primary passage to show that if the pressure ratio across the primary-jet nozzle corresponds to sonic velocity, either raising or lowering p_2 will decrease the thrust.

It is therefore concluded that for a specified mixing-pressure rise, less thrust will be obtained from a compressible fluid than from an incompressible fluid.

Mixing-pressure rise. - The momentum equation, equation (5), is valid for both incompressible and compressible fluids. The determination of the exit velocity from the mixing zone V_3 is more difficult in the case of the compressible fluid because the density variation must be included in equation (6). In computing the mixing-pressure rise, the momentum equation, the general energy equation, and the equation of continuity for the flow of gases were solved simultaneously by numerical methods.

Results of computation of the mixing-pressure rise of compressible and incompressible fluids are compared in figure 8. It may be seen that the incompressible fluid produces the highest mixing-pressure rise at any specified velocity ratio V_s/V_p. The difference is small with a velocity ratio of 0, but becomes greater at high values of V_s/V_p. For example, the pressure rise becomes 0 at a velocity ratio of 1.0 for an incompressible fluid, but for a compressible fluid with a primary-pressure ratio of 10 the mixing-pressure rise becomes 0 at a velocity ratio of only 0.195.

The compressible nature of the fluid has a further effect upon mixing-pressure rise because the relation between the velocity-head ratio and the velocity ratio is influenced. The velocity head for a compressible fluid is greater than for an incompressible fluid, as may be seen in figure 9. This figure shows that at the value of $\frac{\frac{1}{2}\rho V^2}{p}$ of 5, the velocity head for a compressible fluid is 4.4 times the velocity head for an incompressible fluid. Consequently, when the mixing-pressure rise for compressible and incompressible fluids is compared at specified velocity-head ratios, the effect of compressibility is emphasized even more than shown in figure 8. Figure 10 shows that the compressibility indicated by increasing pressure ratio P_p/p_2 reduces the mixing-pressure rise at all values of velocity-head ratio. The conclusion is therefore made that, at least for the case of primary and secondary streams of equal density at the beginning of mixing, the mixing-pressure rise is greater for an incompressible fluid than for a compressible fluid.

Ejector thrust. - The preceding analysis has shown that at a specified pressure drop through the secondary passage the thrust with a compressible fluid is less than the thrust with an incompressible fluid. Furthermore, at a specified velocity-head ratio the mixing-pressure rise is less with a compressible fluid than with an incompressible fluid. Consequently, the ejector thrust will be less with a compressible fluid than with an incompressible fluid. This comparison is illustrated in figure 11, where it is shown that at a primary-pressure ratio P_p/p_2 of 1.2, the ejector thrust is in close agreement for both compressible and incompressible fluids, but at a primary-pressure ratio of 5 the compressible fluid produces less thrust than an incompressible fluid.

SUMMARY OF ANALYSIS

An analysis of the thrust of air ejectors, made chiefly for the purpose of exploring and illustrating the principles of ejector operation, has indicated that the thrust augmentation of a primary jet with incompressible fluids is created largely by pressure forces on the surface of a converging secondary passage preceding the mixing zone. These pressure forces, caused by the static-pressure rise that results from the mixing of fluid streams of different velocities, decrease with increase in secondary velocity. The ratio of the densities of the secondary and primary fluids influences the ejector thrust, the greatest thrust being obtained with equal densities in the primary and secondary streams.

A simple investigation of the performance of ejectors with compressible fluids indicates that mixing-pressure rise and thrust are less with compressible fluids than with incompressible fluids when the ejector is operating at a specified velocity-head ratio at the entrance to the mixing zone.

Lewis Flight Propulsion Laboratory,
National Advisory Committee for Aeronautics,
Cleveland, Ohio, January 11, 1949.

APPENDIX A

SYMBOLS

The following symbols are used in this report:

A	cross-sectional area of passage, sq ft
A_m	mixing-zone cross-sectional area, sq ft
A_{max}	cross-sectional area of diffuser or nozzle A_4 or of mixing zone A_m, whichever is greater, sq ft
A_p	primary-nozzle cross-sectional area, sq ft
A_1	secondary-passage area of ejector entrance, sq ft
A_4	area of exit diffuser or nozzle, sq ft
dA	elemental frontal area of ejector, sq ft
C	constant of integration
F_e	axial force on ejector entrance, lb
F_d	axial force on exit diffuser or nozzle, lb
F_n, F_b	components of force on primary passage, lb
F_p	force on primary-jet passage, lb
$F_{p,f}$	jet thrust of primary-jet engine when ejector is not used, lb
ΔF_p	change in force on primary-jet passage resulting from presence of ejector, lb
P_p	primary total pressure, lb/sq ft
P_s	secondary total pressure, lb/sq ft
P_3	total pressure at exit of mixing zone, lb/sq ft
p	internal pressure on elemental area, lb/sq ft
p_2	primary-nozzle-exit static pressure, lb/sq ft
p_3	mixing-zone-exit static pressure, lb/sq ft

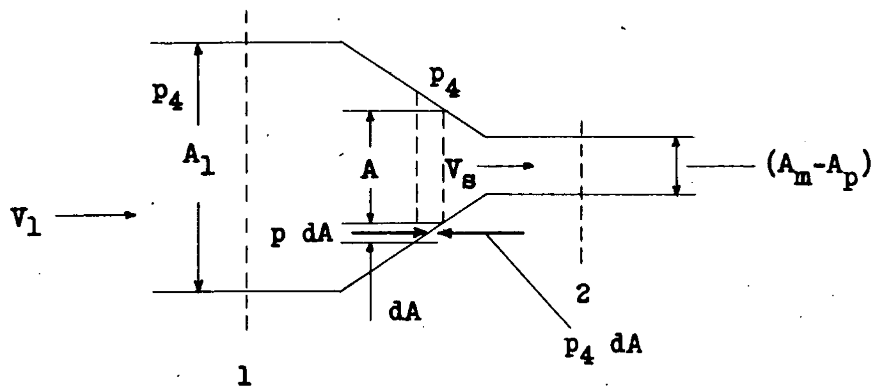
p_4	ejector-exit static pressure (equals ambient pressure), lb/sq ft
$\dfrac{P_s-p_2}{P_p-p_2}$	ratio of velocity heads in secondary and primary streams at mixing-zone entrance
S	augmentation
V	velocity of fluid at position in passage where cross-sectional area is A, ft/sec
V_p	primary velocity at mixing-zone entrance, ft/sec
V_s	secondary velocity at mixing-zone entrance, ft/sec
V_1	fluid velocity at entrance to ejector, ft/sec
V_3	mixing-zone-exit fluid velocity, ft/sec
W_p	weight flow of primary fluid of jet engine with ejector, lb/sec
$W_{p,f}$	weight flow of primary fluid of jet engine without ejector, lb/sec
γ	ratio of specific heats
ρ_p	primary density, slugs/cu ft
ρ_s	secondary density, slugs/cu ft

APPENDIX B

THRUST ON A CONVERGING DUCT

Incompressible Fluid

The thrust on a converging duct is found by integrating the pressures over its surfaces and computing the ratio of entrance area to exit area required to create the pressure differential p_4-p_2. This duct is assumed to be submerged in a fluid at static pressure p_4 and to discharge to a lower static pressure p_2. The following sketch indicates the elements used in the integration:



The thrust on the elemental frontal area dA is

$$dF_e = (p_4-p)\ dA$$

$$P_s = p + \frac{1}{2}\rho_s V^2 = p_2 + \frac{1}{2}\rho_s V_s^2 = p_4 + \frac{1}{2}\rho_s V_1^2$$

$$VA = V_1 A_1 = V_s (A_m-A_p)$$

$$V = \frac{V_s (A_m-A_p)}{A}$$

Hence

$$p = p_2 + (P_s-p_2) - (P_s-p_2)(A_m-A_p)^2 A^{-2}$$

Therefore

$$dF_e = \left[p_4-p_2 - (P_s-p_2) + (P_s-p_2)(A_m-A_p)^2 A^{-2}\right] dA$$

Integration yields

$$F_e = (p_4-P_s) A - (P_s-p_2)(A_m-A_p)^2 A^{-1} + C$$

The limits of integration are (A_m-A_p) and A_1. Values of A and A_1 are found as follows:

$$V_1 A_1 = V_s (A_m-A_p)$$

Hence

$$\frac{A_1}{(A_m-A_p)} = \sqrt{\frac{P_s-p_2}{P_s-p_4}}$$

Substitution of this value of A_1 to evaluate the constant of integration results in the following expression for F_e:

$$\frac{F_e}{P_s (A_m-A_p)} = \left[\left(1 - \frac{p_4}{P_s}\right)^{\frac{1}{2}} - \left(1 - \frac{p_2}{P_s}\right)^{\frac{1}{2}}\right]^2$$

This equation is reducible to the form used in this report (equation (1)):

$$\frac{F_e}{(P_s-p_4)(A_m-A_p)} = \left[1 - \left(1 + \frac{p_4-p_2}{P_s-p_4}\right)^{\frac{1}{2}}\right]^2 \tag{B1}$$

In the derivation of this equation, the roots of all contributing equations were selected when they were applicable to flow from station 1 to station 2 in figure 1.

Compressible Fluid

Thrust on a converging duct handling a compressible fluid is computed by the same basic process of integration of surface pressures. The relations among velocity, pressure, and area are more complicated than in the case of an incompressible fluid. The integration was therefore performed by numerical means. This integration is expressed by the following equation:

$$\frac{F_e}{P_s\ (A_m - A_p)} = \int_{(A_m - A_p)}^{A_1} \left(\frac{p_4}{P_s} - \frac{p}{P_s}\right) \frac{dA}{(A_m - A_p)}$$

where the relation between area of passage and pressure in the isentropic flow of gas is

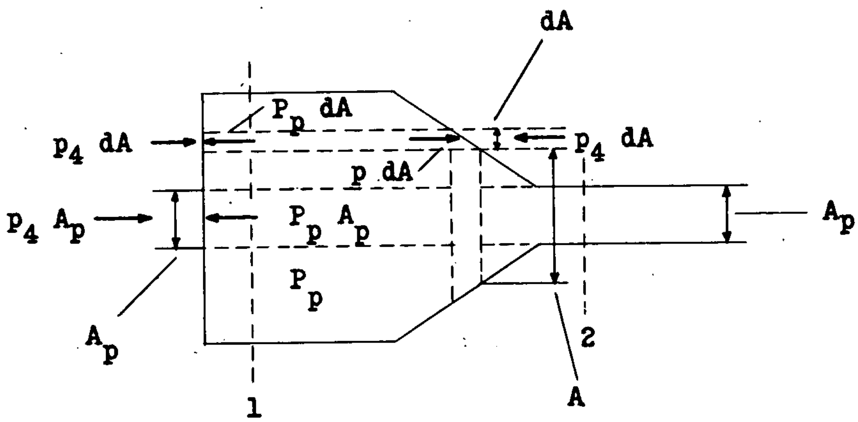
$$\frac{A}{A_m - A_p} = \left[\frac{\left(\frac{p_2}{P_s}\right)^{\frac{2}{\gamma}} - \left(\frac{p_2}{P_s}\right)^{\frac{\gamma+1}{\gamma}}}{\left(\frac{p}{P_s}\right)^{\frac{2}{\gamma}} - \left(\frac{p}{P_s}\right)^{\frac{\gamma+1}{\gamma}}}\right]^{\frac{1}{2}}$$

The thrust parameters thus obtained are shown in figure 12.

APPENDIX C

EFFECT OF EXHAUST PRESSURE ON THRUST OF PRIMARY JET WITH MAINTAINED PRIMARY TOTAL PRESSURE

The effect of the reduction of pressure at the exhaust of the primary jet engine is found by integrating the pressures over the surfaces of the engine. The following sketch is referred to in this derivation:



The surfaces are divided into two regions. One region is an area equal to the nozzle area A_p and is on the wall of the primary engine, but opposite the nozzle. The thrust on this area is

$$F_n = (P_p - p_4)\ A_p$$

The thrust on an elemental area dA of the right wall and a corresponding area on the left wall is

$$dF_b = (P_p - p_4 + p_4 - p)\ dA = (P_p - p)\ dA \qquad \text{(C1)}$$

But

$$p = P_p - \frac{1}{2}\rho_p\ V^2$$

and

$$V = \frac{V_p\ A_p}{A}$$

hence

$$p = P_p - \frac{1}{2} \rho_p \left(\frac{A_p}{A}\right)^2 V_p^{\,2}$$

but

$$V_p^{\,2} = \frac{2\ (P_p - p_2)}{\rho_p}$$

therefore

$$p = P_p - \left(\frac{A_p}{A}\right)^2 (P_p - p_2)$$

then

$$dF_b = P_p\ dA - P_p\ dA + (P_p - p_2)\ A_p^{\,2}\ A^{-2}\ dA$$

Integrating between limits of A_p and infinity yields

$$F_b = (P_p - p_2)\ A_p$$

The total thrust F_p is

$$F_p = F_n + F_b$$

$$= 2\ (P_p - p_4)\ A_p + (p_4 - p_2)\ A_p \qquad \text{(C2)}$$

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2. Ellerbrock, Herman H., Jr.: General Treatment of Compressible Flow in Ejectors and Example of Its Application to Problem of Effect of Ejector Addition on Thrust of Jet-Propulsion Units. NACA RM L6L23, 1947.
3. Clegren, William A.: Advances in Thrust Augmentation for Radial Engine Installations. SAE Quarterly Trans., vol. 2, no. 1, Jan. 1948, pp. 60-70.

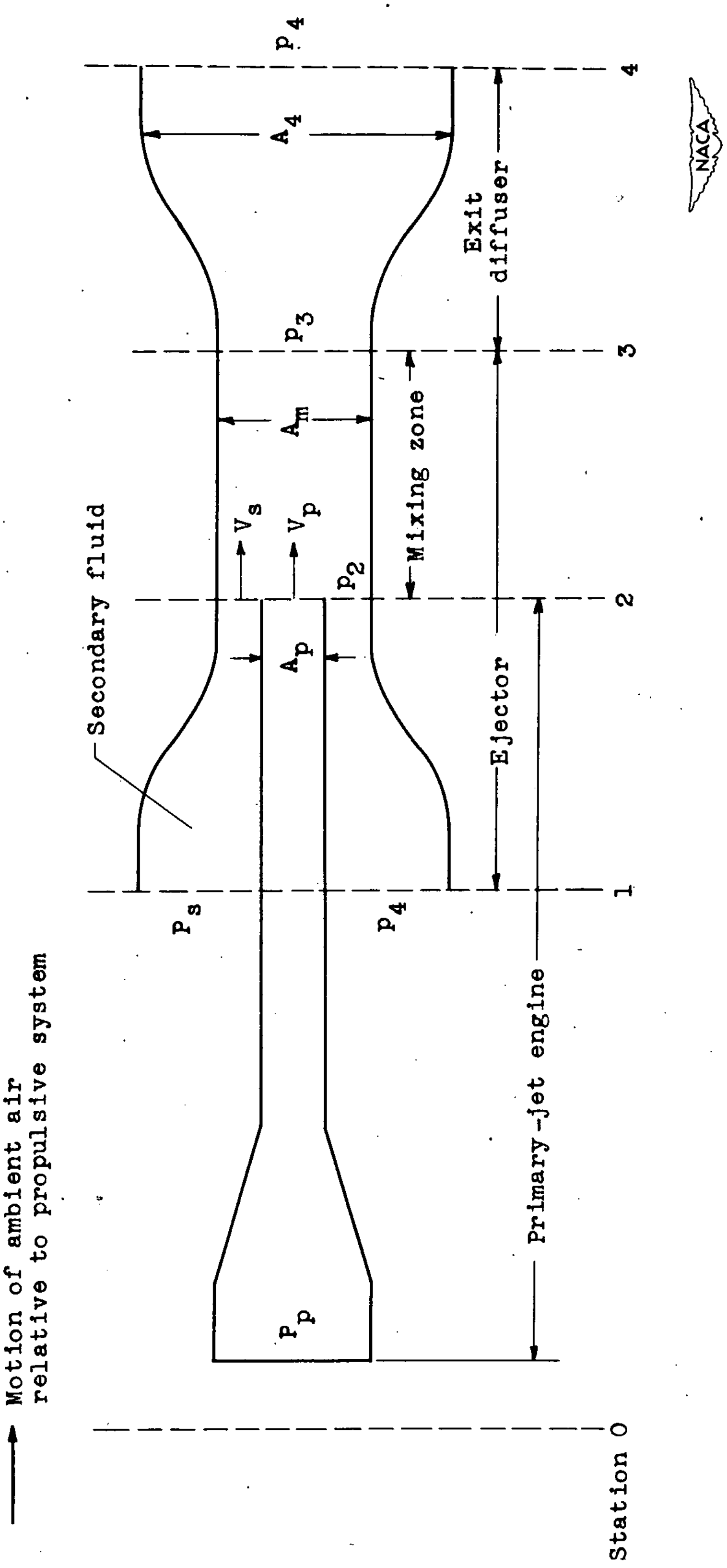

Figure 1. - Air-ejector configuration and nomenclature used in analysis.

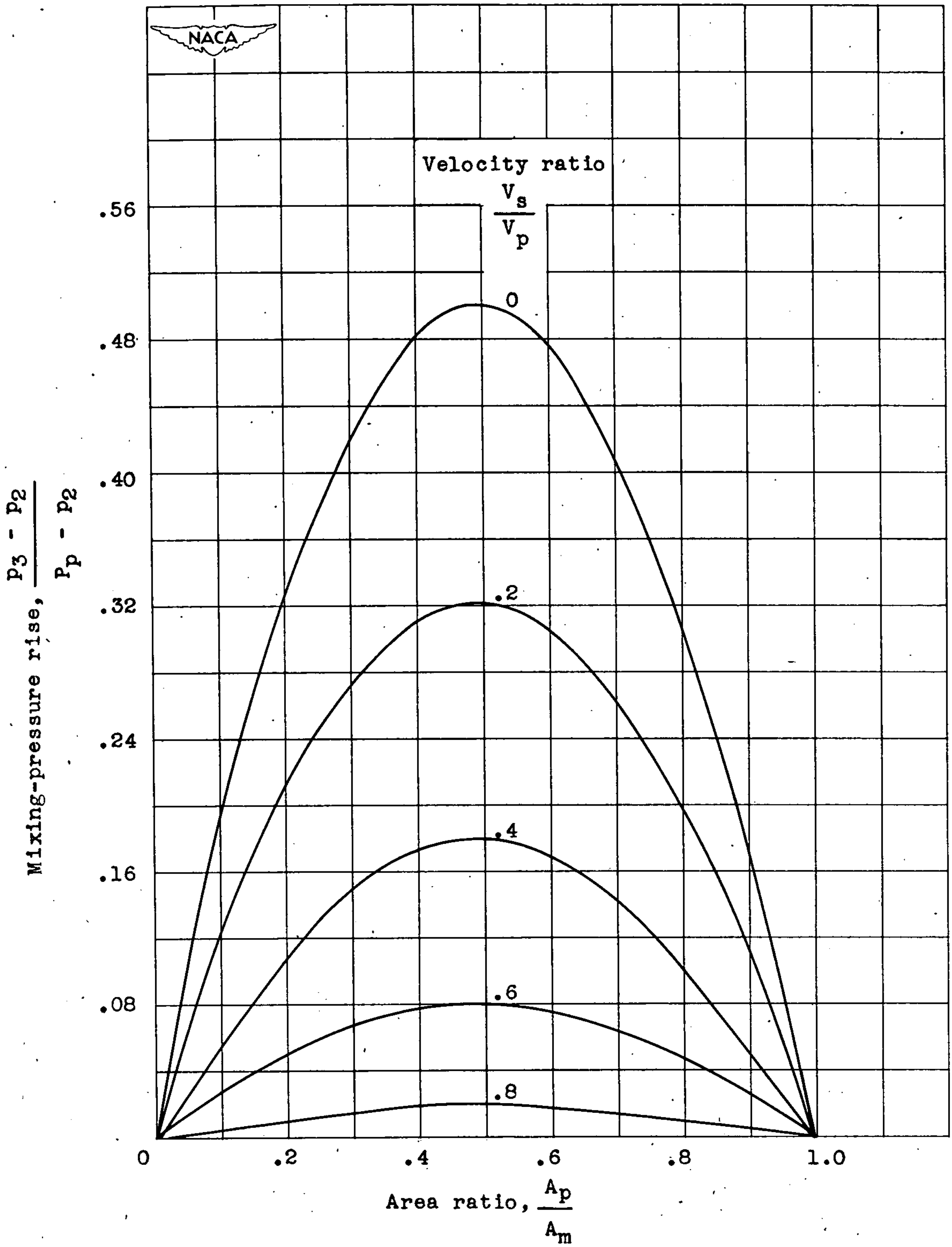

Figure 2. - Pressure rise resulting from mixing of two streams of incompressible fluids having the same densities.

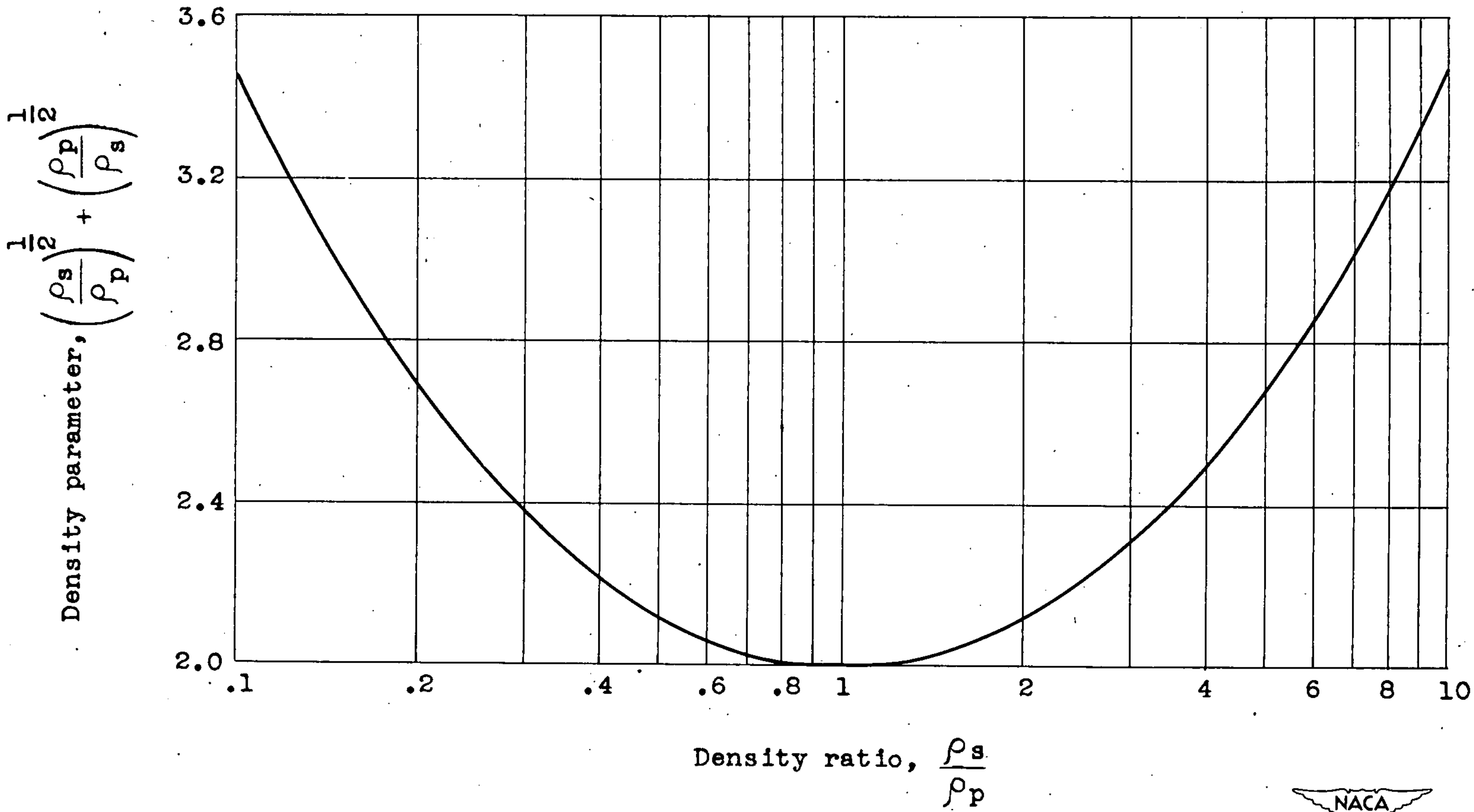

Figure 3. - Relation between density ratio and density parameter.

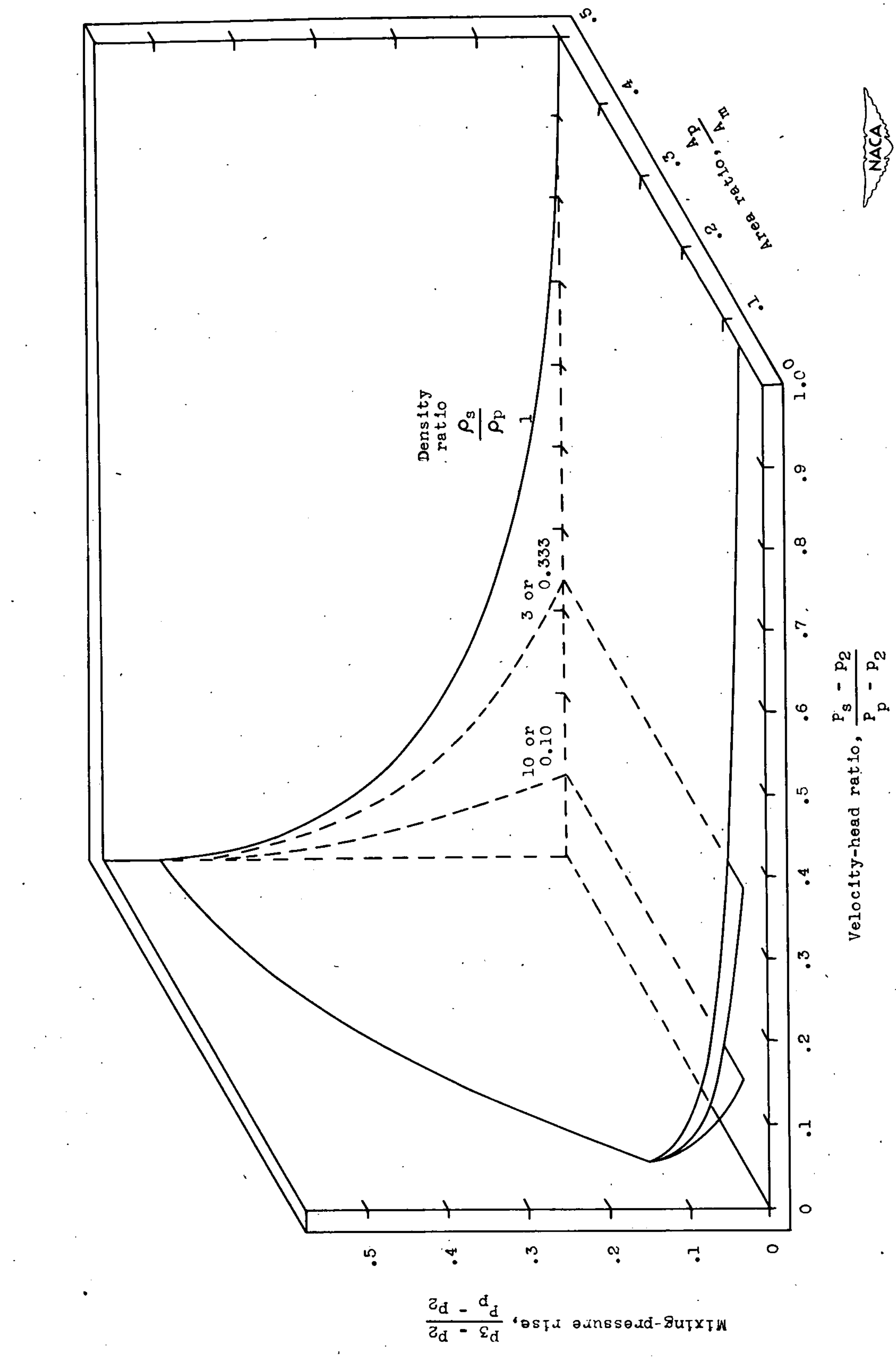

Figure 4. - Effects of density ratio, velocity-head ratio, and area ratio on mixing-pressure rise of two streams of incompressible fluid.

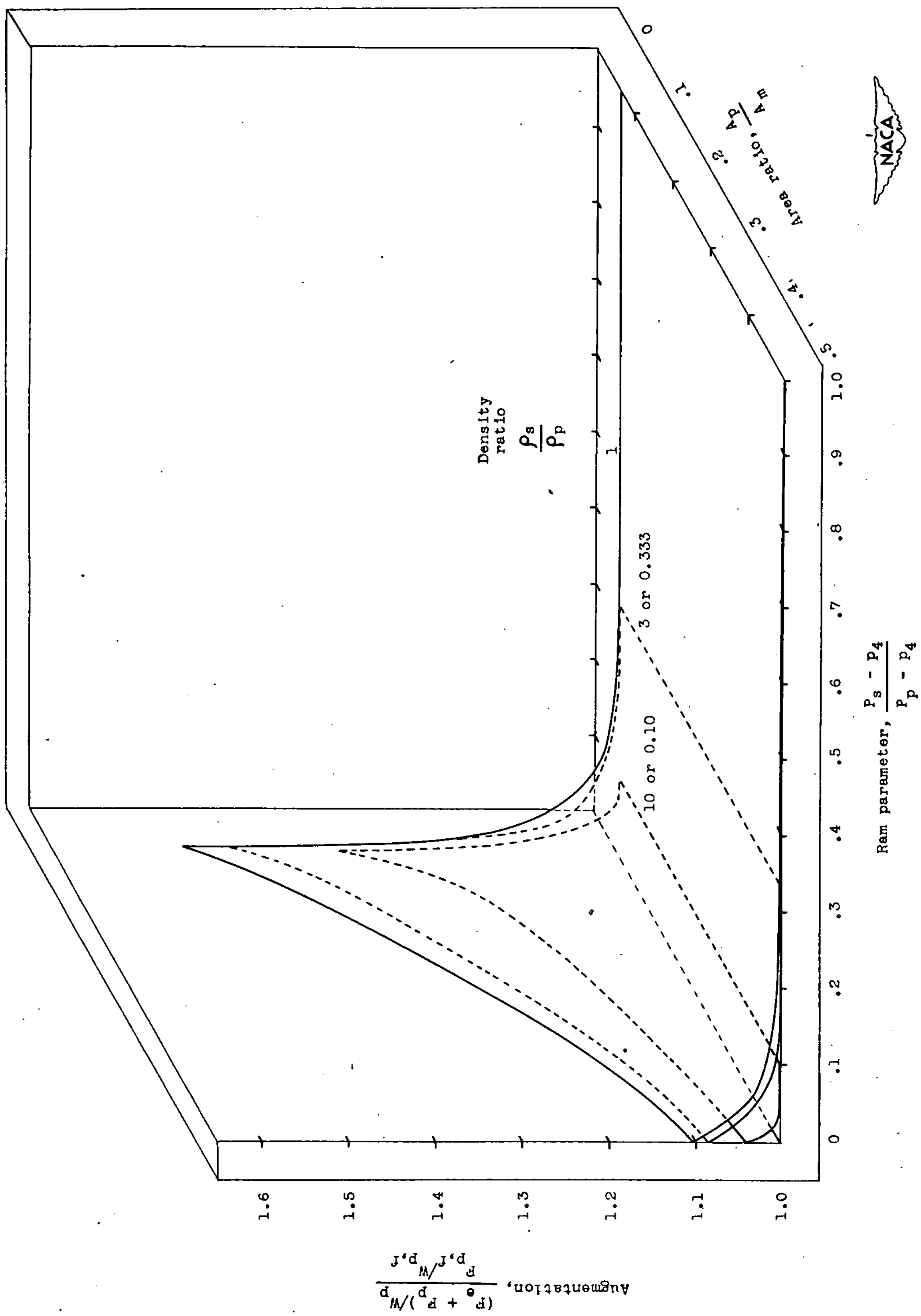

Figure 5. - Thrust augmentation caused by air ejectors attached to jet engine. Incompressible fluids assumed; no exit diffuser.

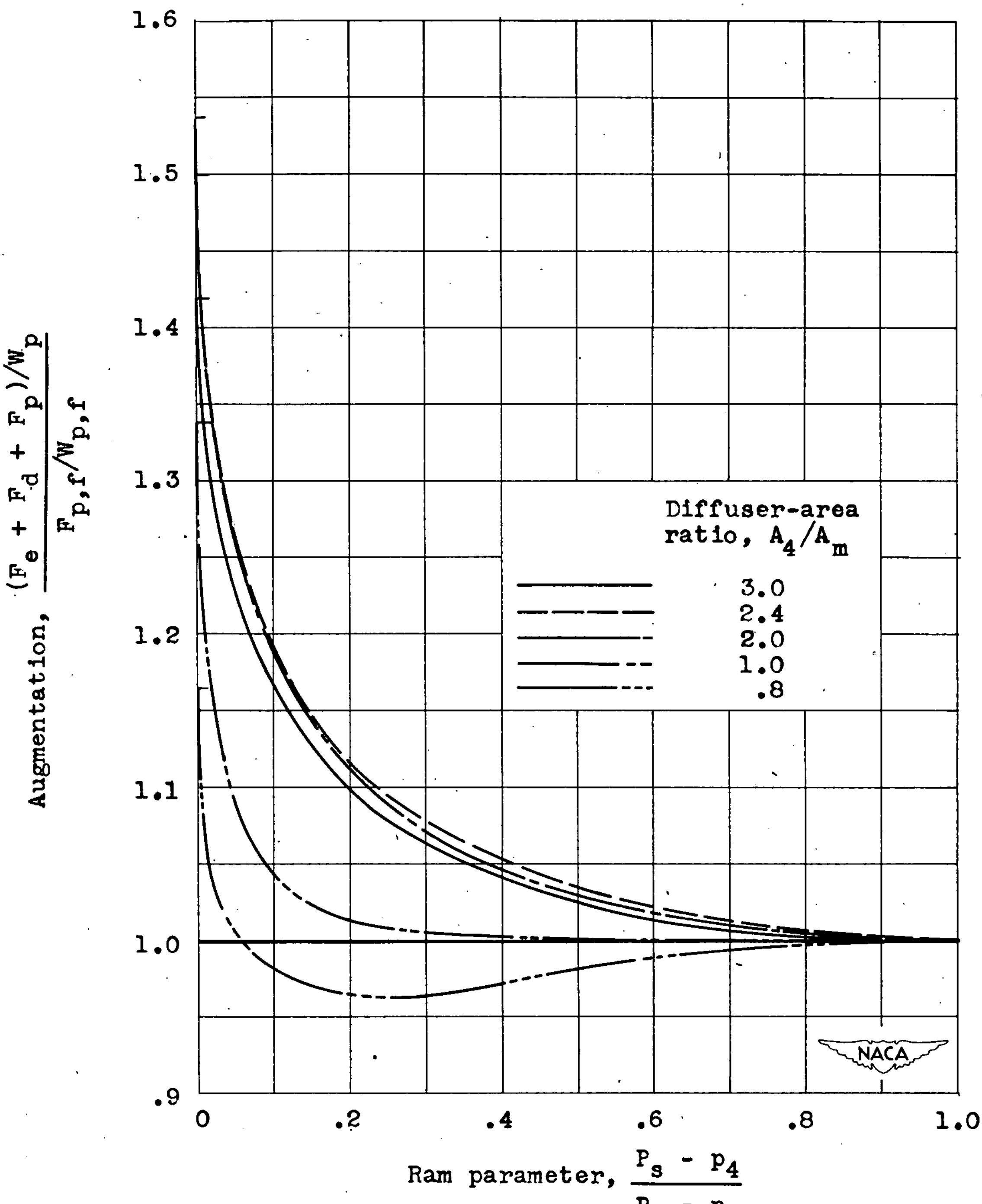

Figure 6. - Influence of exit-diffuser-area ratio A_4/A_m on augmentation of ejectors handling incompressible fluids. Ratio of densities of primary and secondary fluids, 1.0; A_p/A_{max}, 0.15.

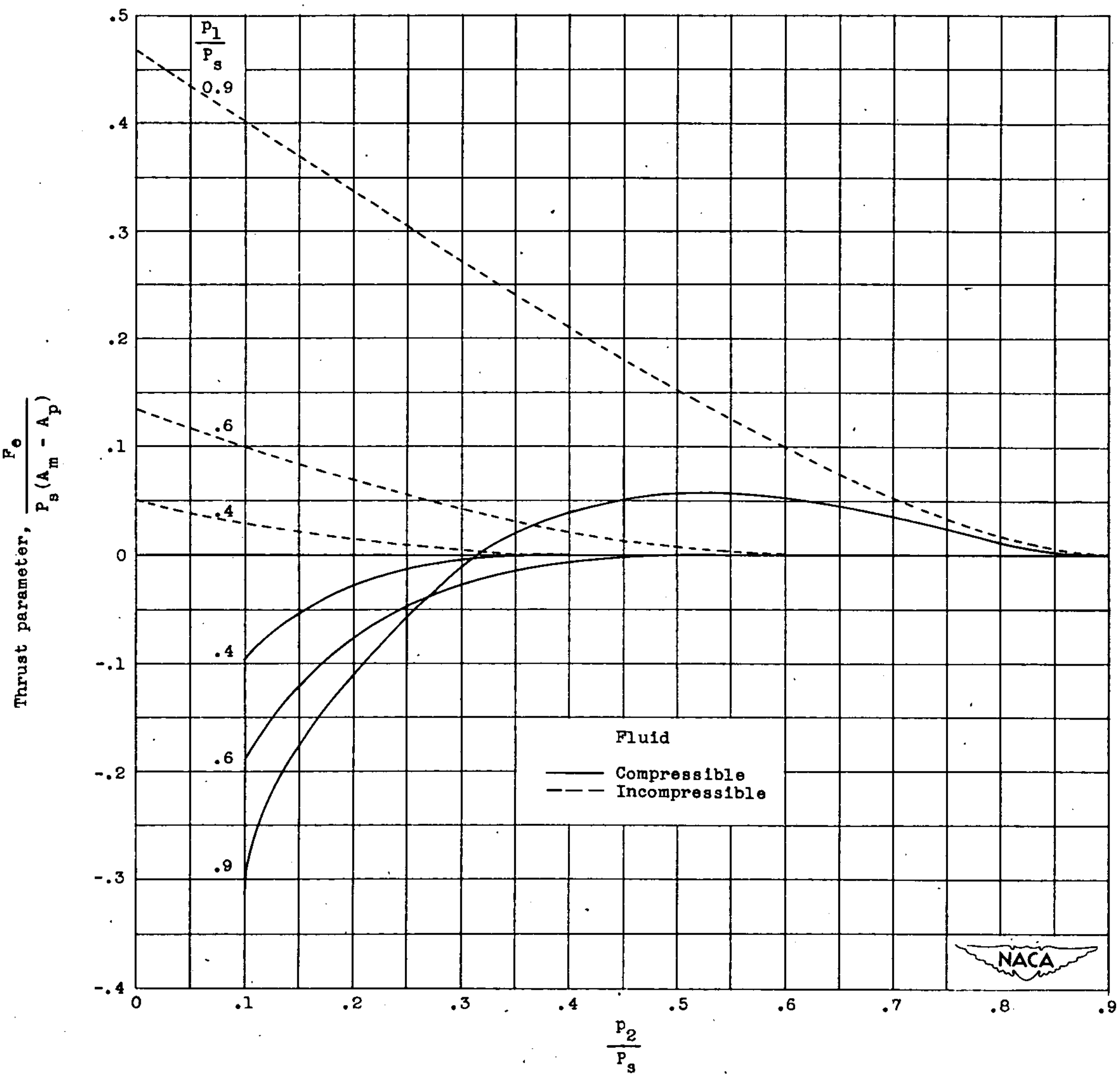

Figure 7. - Comparison of thrusts on converging ducts handling compressible and incompressible fluids. Ratio of specific heats for compressible fluid, 1.4; area ratio A_p/A_m, 0.0625; density ratio ρ_s/ρ_p, 1.0.

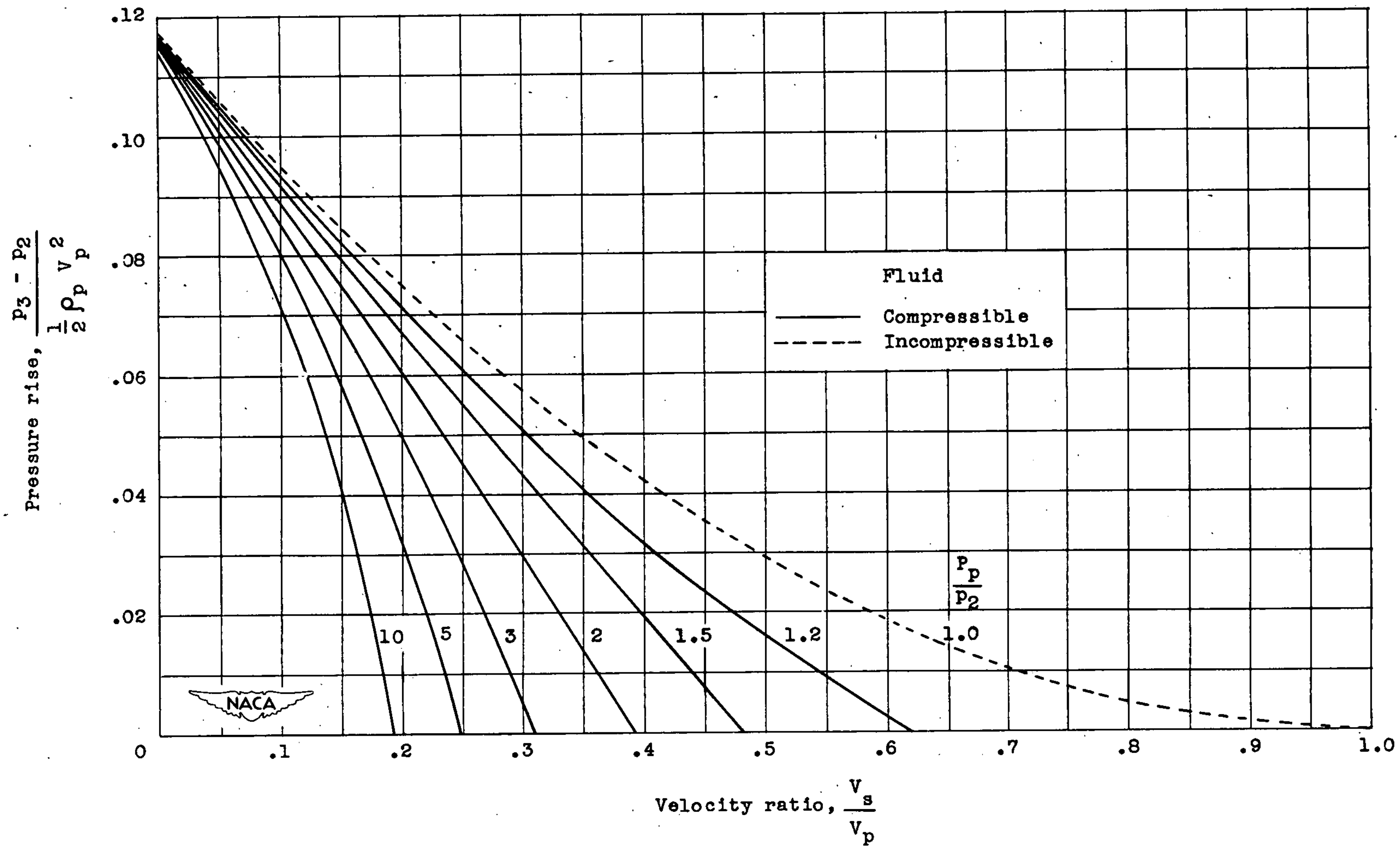

Figure 8. - Comparison of effects of velocity ratio on pressure rise in mixing for compressible and incompressible fluids. Area ratio A_p/A_m, 0.0625; density ratio ρ_s/ρ_p, 1.0.

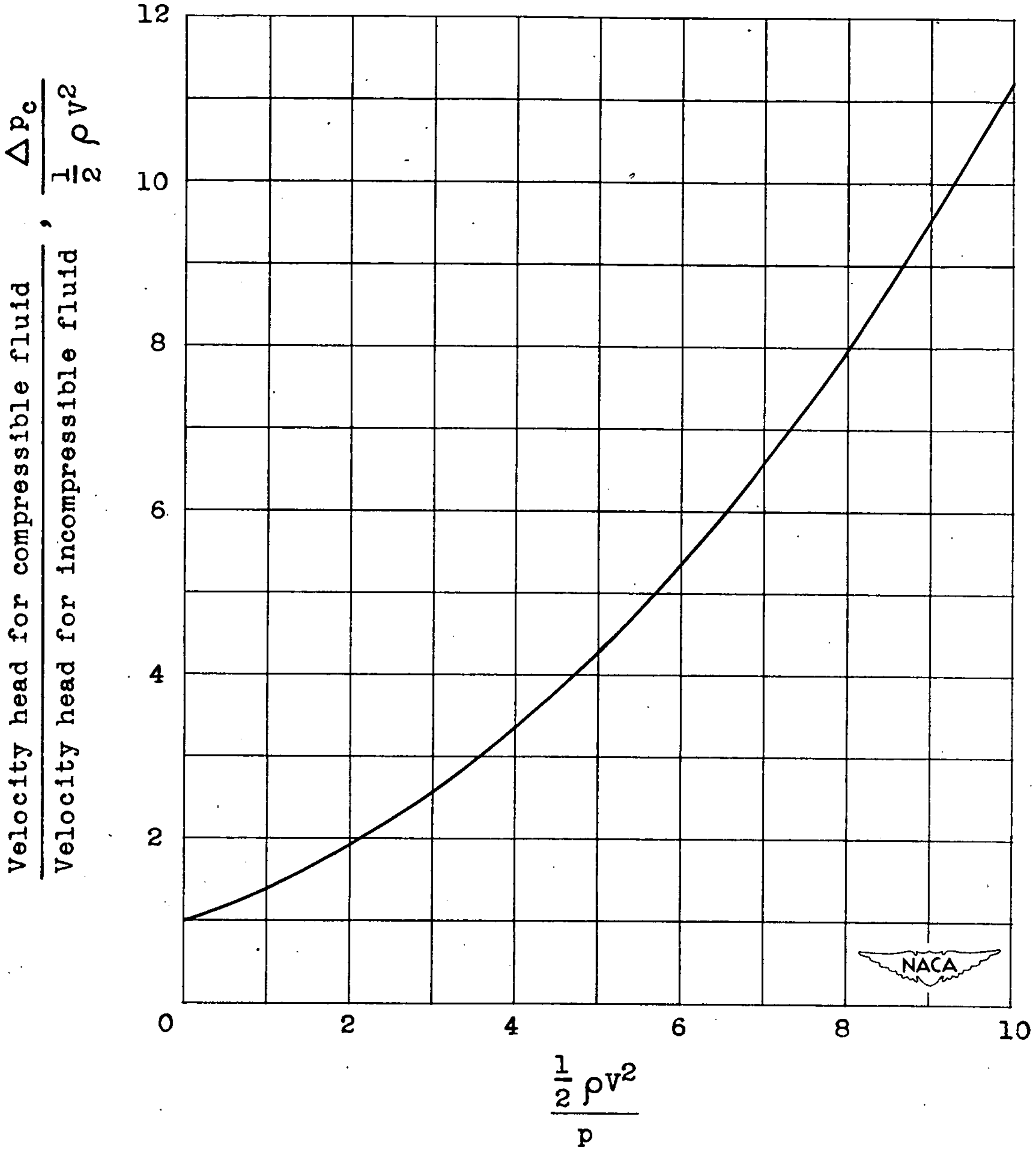

Figure 9. - Relation of velocity head in compressible to that in incompressible fluids.

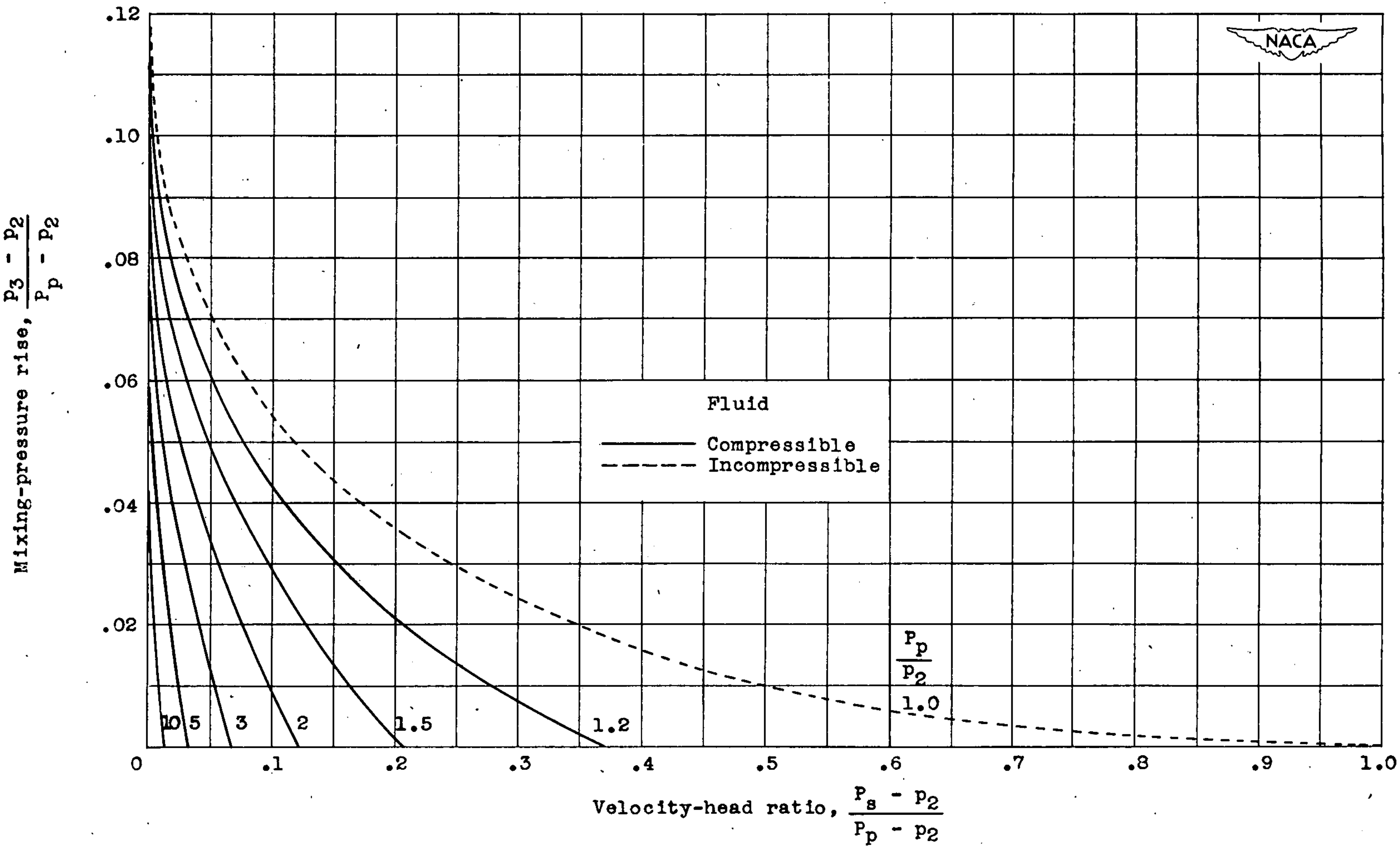

Figure 10. - Comparison of effects of ratio of velocity heads in secondary and primary streams on mixing-pressure rise for compressible and incompressible fluids. Area ratio A_p/A_m, 0.0625; density ratio ρ_s/ρ_p, 1.0.

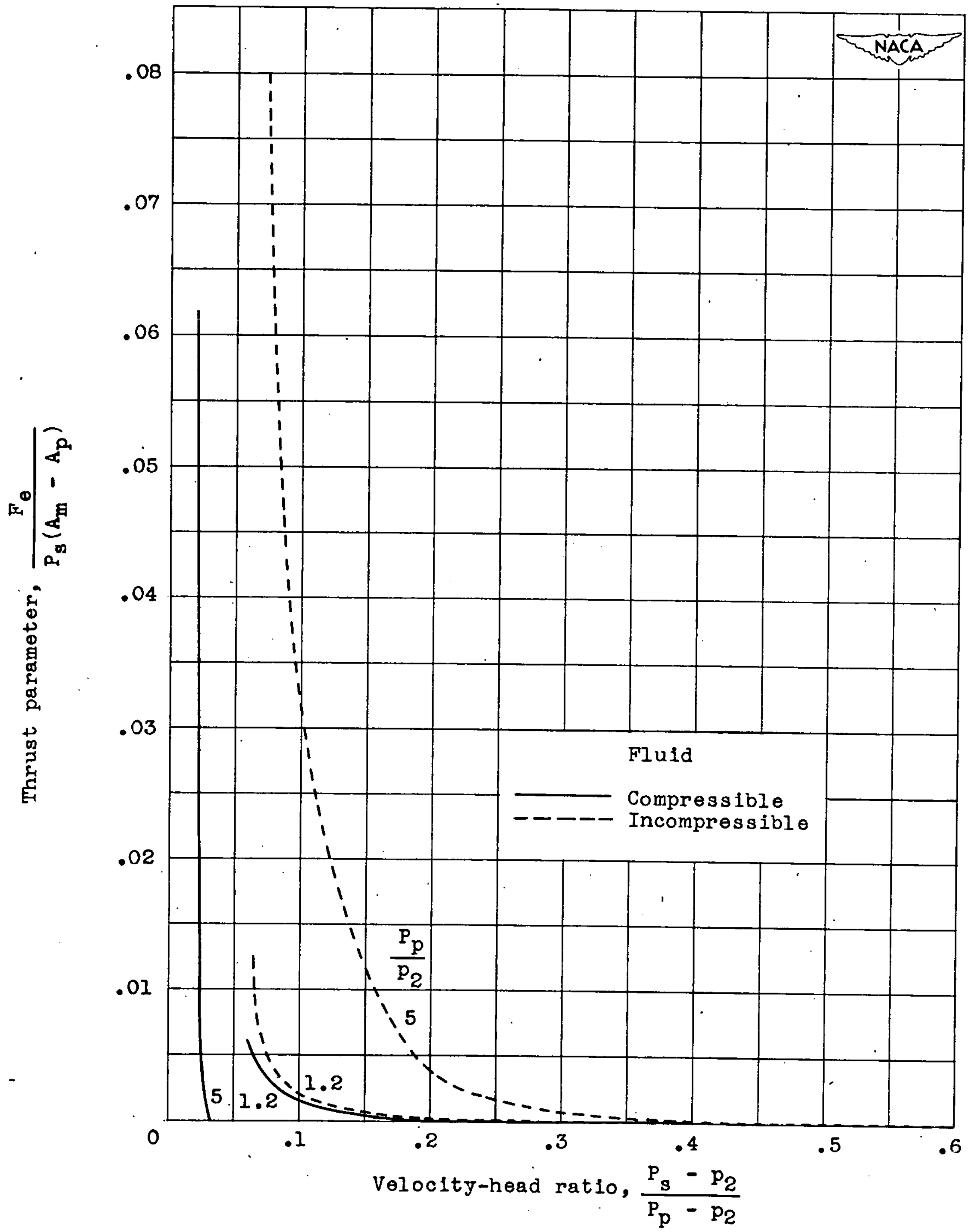

Figure 11. - Comparison of thrusts on ejectors handling compressible and incompressible fluids. Area ratio A_p/A_m, 0.0625; density ratio ρ_s/ρ_p, 1.0.

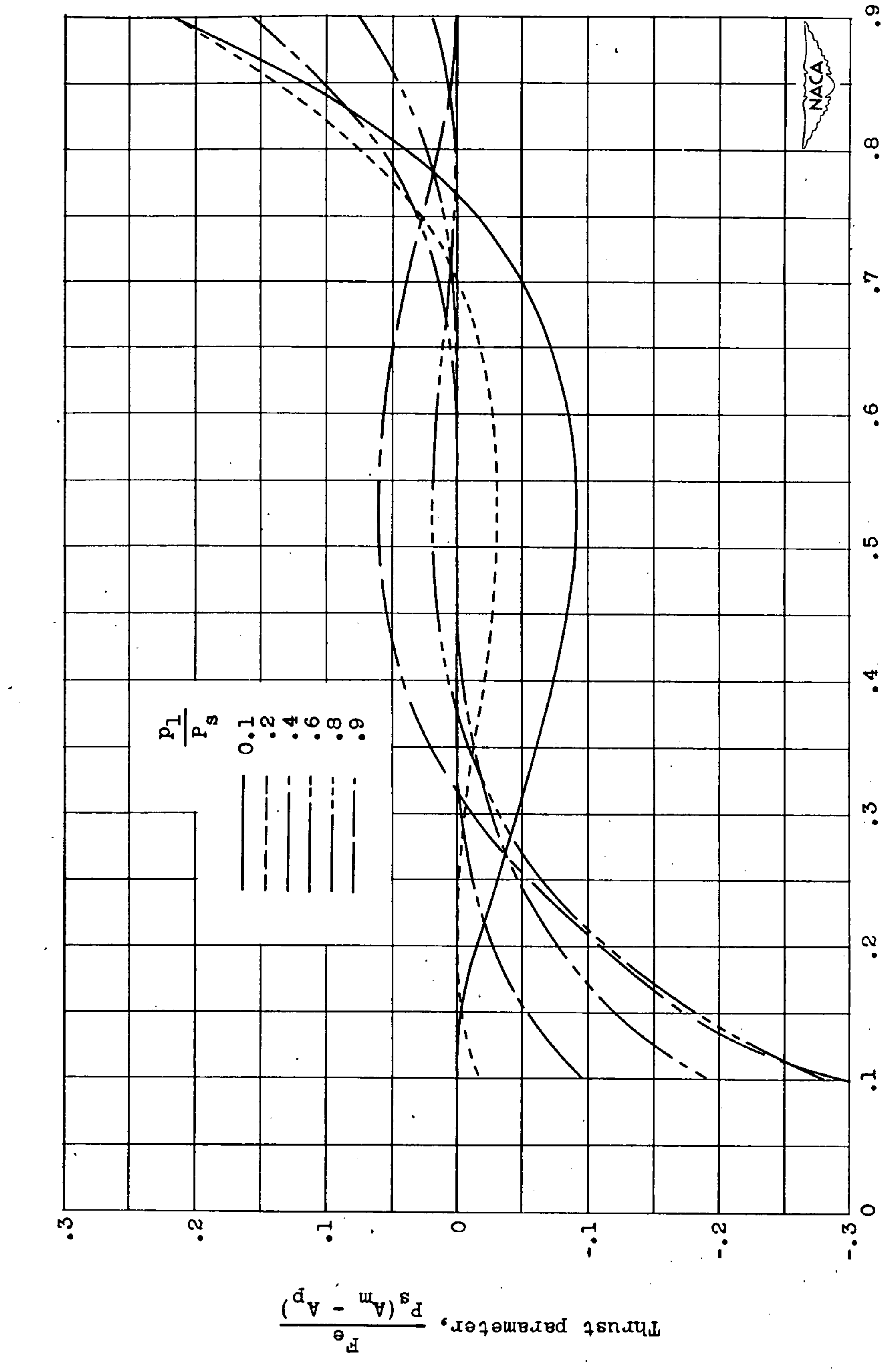

Figure 12. - Axial force on frictionless duct handling compressible fluid. Duct matched to entrance and exit conditions. Area ratio A_p/A_m, 0.0625; density ratio ρ_s/ρ_p, 1.0.